EFFECT OF PULSATILE FLOW OF BLOOD THROUGH MULTIPLE STENOSED ARTERY IN THE PRESENCE OF MAGNETIC FIELD AND BODY ACCELERATION.

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ABSTRACT

Pulsatile flow of blood under the influence of externally imposed magnetic field and periodic body acceleration through a multistenosed artery is studied. In this problem a Mathematical model is developed by treating blood as a non Newtonian fluid characterized by the Casson fluid. The pulsatile is analyzed by considering a periodic pressure gradient which is a function of time. In this analysis we try to find the computational result by using the Perturbution analysis, assuming that the Womersely frequency parameter is very small. The effect of pulsatility and body acceleration have been discussed with the help of graphs.

Keywords: Pulsatile flow, Casson fluid, Body acceleration, Stenosed artery, Perturbution analysis.

I. INTRODUCTION

The study of blood flow through a multi stenosed artery is very important because of the fact that cause and development of many cardiovascular diseases are related to the nature of blood movement and the mechanical behavior of the blood vessel walls. A stenosis is defined as a partial occlusionof the blood vessel due to the accumulation of cholesterols and fats and the abnormal growth of tissues. The stenosis is one of the most frequently anomaly in blood circulation. Once the constrictionis formed , the blood flow is significantly altered and fluid dynamical factors play important role as the stenosis continues to enlarge leading to the development of cardiovascular disease such as heart attack and stroke etc.

In our daily life we often face some external body acceleration, such as travelling in vehicles, aircrafts, running with high velocity etc. these types of situations undoubtly affects the normal flow of blood which lead to headache, vomiting tendency, loss of vision and abnormality in pulse rate etc. Many researchers have studied blood flow in the artery by considering blood as Newtonian or non-Newtonian fluids. Since blood is a suspension of red cells in plasma. It behaves like a non- Newtonian fluid at low shear rate.

Various Mathematical models have been investigated to explore the behavior of blood flow under the influence of external acceleration. So, the body acceleration plays a important role in blood flow in arteries. Such body acceleration is usually caused unintentionally. Sud et. al. (1985) made an analysis of blood flow under the time

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dependent acceleration and obtained a result which showed that the high blood velocities and high shear rate are capable of harming the circulation which produced under the influence of such time dependent acceleration. Sud et. al. (1986) againworked on the analysis of blood flow through a model of the human arterial system under the periodic condition and showed that body acceleration increases the flow rate. Chturani and Palanisamy (1990) analyzed the pulsatile flow of blood under the influence of periodic body acceleration by assuming blood as a power low fluid by using finite difference method. Majhi and Nair(1994) studied the pulsatile flow of blood under the influence of body acceleration treating blood as third grade fluid. Sarojani and Nagrani (2002) studied the flow of Casson fluid in a tube filled with a porous medium under the periodic body acceleration with application on artificial organs. Prashant et. al. (2007) studied the effect of body acceleration on pulsatileflow of non-Newtonian fluid through a stenosed artery and observed that all the instantaneous flow characteristics are affected by the application of body acceleration. Divyajyoti and Uday (2009) worked on the pulsatile flow of blood in a constricted artery with body acceleration and observed that the velocity and flow rate increases but effective viscosity decreases, due to slip wall. Kumar and Dixit (2010) studied Mathematical model for effect of body acceleration on blood flow in the time dependent stenosed artery. Biswas (2011) analyzed steady flow of blood through a stenosed artery: a non Newtonian fluid model. Recently Bali and Awasthi (2012) discussed a Casson fluid model for multiple stenosis artery in presenceof magnetic field.

In this model we consider the problem of Bali and Awasthi (2012) with the introduction of pulsatile flow with body acceleration under the same condition. The aim of present investigation is to study the effect of pulsatile flow and body acceleration on velocity profile, the volumetric flow rate, the wall shear stress and resistance to the blood flow.

Mathematical Formulation:-Let us consider the pulsatile flow of blood through a multiple stenosed artery under the influence of external applied uniform transverse magnetic field and body acceleration. The geometry of stenosis is shown in figure 1. We have taken some assumptions for solving the problem.

1) Let us take the flow of blood as axially symmetric and fully developed(i.e.$v_r = v_\theta = 0$, flow in z- direction only). This is entirely reasonable and reinforces the fact that in unsteady state incompressible flow in a circular tube of uniform cross section. The velocity does not changein the direction of flow, except near the entrance and exit regions.
2) Consider blood as casson fluid(non- Newtonian) and magnetic fluid as red blood cells is a major biomagnetic substance and blood flow may be influenced by the magnetic field.
3) Consider the transverse magnetic field. Since the biomagnetic fluid(blood) is subjected to magnetic field, the action of magnetization will introduce rotational motion to orient the magnetic fluid particle with the magnetic field.

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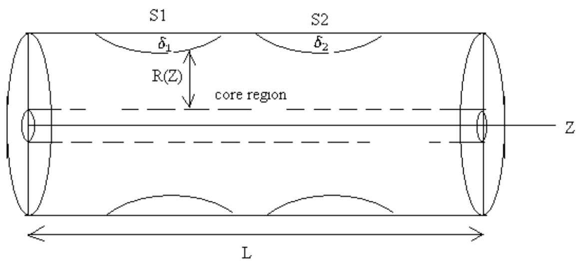

Figure :-1 geometry of the multiple stenosed artery.

The geometry of the multiple stenosed artery in non dimentional form is given by

$$R(z) = \begin{cases} 1 - C\left[S_L^{\Gamma-1}\{z-(b_1+b_2)\} - \{z-(b_1+b_2)\}^{\Gamma}\right] & for\ (b_1+b_2) \le z \le (b_1+b_2)+S_L \\ 1 & otherwise \end{cases} \quad \text{--(`1)}$$

$$where\ C = \frac{\delta}{R_0(S_L)^{\Gamma}} \quad ----(2)$$

Where δ is maximium height of stenosis

$$z = (b_1+b_2) + \frac{S_L}{\Gamma^{\Gamma-1}} \quad ------(3)$$

Where $\Gamma \geq 2$ is the parameter for determining the shape of steosis. $\frac{\delta}{R_0} \ll 1.$

The specified momentum equation for the flow in cylindrical coordinate system is given by

$$\rho'\frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial z'} - \frac{1}{r'}\frac{\partial(r'\tau')}{\partial r'} + G(t') + \mu_0 M\frac{\partial H'}{\partial z'} \quad ---(4)$$

Where r' and z' denotes the radial and axial coordinates respectively,

The above assumptions for Navier –Stokes equations is given by μ_0 is magnetic permeability, M is magnetization,H' is magnetic field density and ρ is pressure density of the fluid and $G(t')$ is the periodic body acceleration which is in the axial direction then $G(t')$ is given by

$$G(t) = a_0\cos(\omega_1 t' + \varphi) \quad t' \geq 0 \quad --(5)$$

Where a_0 is its amplitude, $\omega_1 = 2\pi f_1$ where f_1 is frequency and φ is its phase difference. the frequency of body acceleration is assumed to be small so that wave effects can be neglected. the pressure gradient $\frac{\partial p'}{\partial z'}$ is take as :

$$-\frac{\partial p'}{\partial z'} = A_0 + A_1\cos(\omega_2 t + \varphi) \quad t \geq 0 \quad ---(6)$$

where A_0 is the steady state part of pressure gradient,A_1 is the amplitude of the oscillatory part, $\omega_2 = 2\pi f_2$, f_2 being the heart pulse frequency. For casson fluid, the relation between shear stress and shear rate is :

$$-\frac{\partial u'}{\partial r'} = \frac{\left(\sqrt{\tau'} - \sqrt{\tau_0'}\right)^2}{\mu} \quad for\ \tau' \le \tau_0' \quad ---7(a)$$

$$\frac{\partial u'}{\partial r'} = 0, \quad for\ \tau' \geq \tau_0' \quad ---(7(b))$$

Here τ' is the shear stress, τ_0' is the yield stress and μ is the coefficient of viscosity of blood along the axis of symmetry and the axial velocity gradient is vanishing in regions where the shear stress τ' is less than the yield stress τ_0'. thus, when the plug flow is $\tau' \leq \tau_0'$.

The boundary conditions governing the problem as follows:

$u' = 0 \ at\ r' = R'(z') \quad ------(8(a))$

$\tau' is\ finite\ at\ r' = 0 \quad ----(8(b))$

In the core region $u' = u_c'\ at\ r' = R_c' \quad ----(8(c))$

Where u_c' is the core velocity.

Introducing non dimensional variables

$$z = \frac{z'}{R_0}, u = \frac{u'}{\frac{A_0R_0^2}{4\mu}}, R(z) = \frac{R'(z)}{R_0}, r = \frac{r'}{R_0}, t = \omega_2 t', \delta = \frac{\delta'}{R_0},$$

$$\tau = \frac{2\tau'}{A_0R_0}, \theta = \frac{2\tau_0'}{A_0R_0}, \omega = \frac{\omega_1}{\omega_2}, B = \frac{a_0}{A_0}, e = \frac{A_1}{A_1}, \alpha^2 = \frac{R_0^2\omega_2\rho}{\mu}, H = \frac{H'}{H_0} \quad ---(9)$$

Where H_0 is transverse uniform magnetic field the non dimensional momentum equation (4) becomes

$$\alpha^2\frac{\partial u}{\partial t} = 4(1 + ecost) + 4Bcos(\omega t + \varphi)) + \frac{2}{r}\frac{\partial(r\tau)}{\partial r} + F_1\frac{\partial H}{\partial z} \quad ---(10)$$

Where $\alpha^2 = \frac{R_0^2\omega_2\rho}{\mu}$ and $F_1 = \frac{\mu_0H_0M}{\frac{A_0R_0^2}{4}}$

α^2 *is Womersley parameter*

Equation (7(a)) and 7(b) can be written as

$$\tau^{1/2} = \theta^{1/2} + \frac{1}{\sqrt{2}}\left(-\frac{\partial u}{\partial r}\right)^{1/2} \quad if\ \tau \geq \theta \quad ---(11(a))$$

$$\frac{\partial u}{\partial r} = 0 \quad if\ \tau \leq \theta \quad ---(11(b))$$

The boundary conditions 8(a),8(b) and 8(c) reduced to

$u = 0\ at\ r = R(z) \quad -----(12(a))$

$\tau\ is\ finite\ at\ r = 0 \quad ----(12(b))$

In the core region $u = u_c\ at\ r = R_c \quad ----(12(c))$

II. ANALYTICAL SOLUTION OF THE PROBLEM

considering the Womersley parameter to be small ($\alpha << 1$), the velocity u, shear stress τ plug core radius R_P and plug core velocity u_P are expressed in the following form:

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + -- \quad ...(13a)$$

$$\tau(z,r,t) = \tau_0(z,r,t) + \alpha^2\tau_1(z,r,t) + -- \quad ...(13b)$$

$$R_c(z,r,t) = R_{0c}(z,r,t) + \alpha^2 R_{1c}(z,r,t) + -- \quad ...(13c)$$

$$u_c(z,r,t) = u_{0c}(z,r,t) + \alpha^2 u_{1c}(z,r,t) + -- \quad ...(13d)$$

Substituting (13a) and (13b) in equation (10) and equating constant terms and α^2 terms we get

$$\frac{\partial}{\partial r}(r\tau_0) = -2r\left[(1 + ecost) + Bcos(\omega t + \varphi) + F_1 \frac{\partial H}{\partial z}\right] \quad ---(13)$$

$$\frac{\partial}{\partial r}(r\tau_0) = -2r\left[g(t) + F_1 \frac{\partial H}{\partial z}\right] \quad ---(14)$$

$$\frac{\partial u_0}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau) \quad ---(15)$$

Where $g(t) = (1 + ecost) + Bcos(\omega t + \varphi) \quad ---(15a)$

Integrating equation (14) and using boundary conditions 8(a),8(b) and 8(c) we get

$$\tau_0 = -r\left[g(t) + \frac{F_1}{4}\frac{\partial H}{\partial z}\right] \quad ---(15b)$$

Substituting (13a) and (13b) in equation (11a) then we get

$$-\frac{\partial u_0}{\partial r} = 2\left[\theta + \tau_0 - 2\sqrt{\theta\tau_0}\right] \quad ---(16)$$

$$-\frac{\partial u_1}{\partial r} = 2\tau_1\left[1 - \sqrt{\frac{\theta}{\tau_0}}\right] \quad ---(17)$$

Using relation (15a) and (15b) and boundary condition (8a) and (8b) in equation (16) then we obtain the axial velocity in equation (18)

$$u_0 = \left[\left(g(t) + \frac{F_1}{4}\frac{\partial H}{\partial z}\right)(R^2 - r^2) - \frac{8\sqrt{\theta\left(g(t) + \frac{F_1}{4}\frac{\partial H}{\partial z}\right)}}{3}\left(R^{\frac{3}{2}} - r^{\frac{3}{2}}\right) + 2\theta(R - r)\right] \quad --(18)$$

The core velocity can be determined by equation (18):

$$u_{0c} = \left[\left(g(t) + \frac{F_1}{4}\frac{\partial H}{\partial z}\right)(R^2 - R_{0c}^2) - \frac{8\sqrt{\theta\left(g(t) + \frac{F_1}{4}\frac{\partial H}{\partial z}\right)}}{3}\left(R^{\frac{3}{2}} - R_{0c}^{\frac{3}{2}}\right) + 2\theta(R - R_{0c})\right] \quad --(19)$$

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Similarly τ_1, u_1 and u_{1c} can be obtained by using equation (15),(18), and (19) as :

$$\tau_1 = \frac{g'(t)R^3}{8}\left[2\left(\frac{r}{R}\right) - \left(\frac{r}{R}\right)^3 - \frac{8}{21}\frac{\sqrt{\theta}}{\sqrt{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}}\left\{7\left(\frac{r}{R}\right) - 4\left(\frac{r}{R}\right)^{\frac{5}{2}}\right\}\right] \quad ---(20)$$

$$u_1 = \frac{g'(t)R^4}{8}\left[\left(\frac{r}{R}\right)^4 + 4\left(\frac{r}{R}\right)^2 + 3 + \frac{\sqrt{\theta}}{\sqrt{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}}\left\{\frac{16}{3}\left(\frac{r}{R}\right)^2 - \frac{424}{147}\left(\frac{r}{R}\right)^{\frac{7}{2}}\right\} - \frac{1144}{147} + \frac{\theta}{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}\left\{\frac{128}{63}\left(\frac{r}{R}\right)^3 - \frac{64}{9}\left(\frac{r}{R}\right)^{\frac{3}{2}} + \frac{320}{63}\right\}\right] \quad ---(21)$$

$$u_{1c} = \frac{g'(t)R^4}{8}\left[\left(\frac{R_{0c}}{R}\right)^4 + 4\left(\frac{R_{0c}}{R}\right)^3 + 3 + \frac{\sqrt{\theta}}{\sqrt{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}}\left\{\frac{16}{3}\left(\frac{R_{0c}}{R}\right)^2 - \frac{424}{147}\left(\frac{R_{0c}}{R}\right)^{\frac{7}{2}}\right\} - \frac{1144}{147} + \frac{\theta}{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}\left\{\frac{128}{63}\left(\frac{R_{0c}}{R}\right)^3 - \frac{64}{9}\left(\frac{R_{0c}}{R}\right)^{\frac{3}{2}} + \frac{320}{63}\right\}\right] \quad ---(22)$$

$$u = \left[\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)(R^2 - r^2) - \frac{8\sqrt{\theta\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)}}{3}\left(R^{\frac{3}{2}} - r^{\frac{3}{2}}\right) + 2\theta(R - r)\right] + \alpha^2\frac{g'(t)R^4}{8}\left[\left(\frac{r}{R}\right)^4 + 4\left(\frac{r}{R}\right)^2 + 3 + \frac{\sqrt{\theta}}{\sqrt{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}}\left\{\frac{16}{3}\left(\frac{r}{R}\right)^2 - \frac{424}{147}\left(\frac{r}{R}\right)^{\frac{7}{2}}\right\} - \frac{1144}{147} + \frac{\theta}{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)R}\left\{\frac{128}{63}\left(\frac{r}{R}\right)^3 - \frac{64}{9}\left(\frac{r}{R}\right)^{\frac{3}{2}} + \frac{320}{63}\right\}\right] \quad ---(23)$$

The volumetric flow rate Q(t) is given by

$$Q(t) = 4\int_0^R urdr \quad ----(24)$$

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$$Q(t) = R^4\left\{\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)\frac{1}{4}+\frac{4\sqrt{\theta\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)}}{7}+\frac{1}{3}\frac{\theta}{R}+\frac{\alpha^2R^2C}{16}\left[\frac{2}{3}+\frac{120\sqrt{\theta\left(g(t)+\frac{F_1}{4}\frac{\partial H}{\partial z}\right)}}{77\sqrt{R}}+\frac{32}{35}\frac{\theta}{R}\right]\right\}$$

The resistance to flow can be calculated by using the formula

$$\lambda(t) = \frac{P_0 - P_L}{Q} \quad ----(25)$$

Where $P=P_0$ at z=0 and $P=P_L$ at z=L

III. RESULT AND DISCUSSION

This model has been presented to study some aspects of flow of blood through an artery having multiple stenosis. The objective of the present work is to study the effect of body acceleration on the pulsatile flow of blood flow through an artery having multiple stenosis. The results are discussed by computing the flow flow variables at suitable values of θ, B, δ, e and t by fixing other parameter occurred in the flow.

Fig.- 1 Shows the variation of axial velocity with radial coordinate for different values of B(Body acceleration paramenter). It is clear that as B increases, axial velocity increases.

Fig.- 2 Represents the variation of volumetric flow rate with time t for different values of B. it is observed that as B increases volumetric flow rate increases.

IV.CONCLUSION

the pulsatile blood flow through an artery having multiple stenosis have been studied in the presence of externally imposed body acceleration. Results have been shown that all the instantaneous flow characteristics are affected by the application of body acceleration. It is observed that the presence of body acceleration increases the axial velocity and volumetric flow rate.

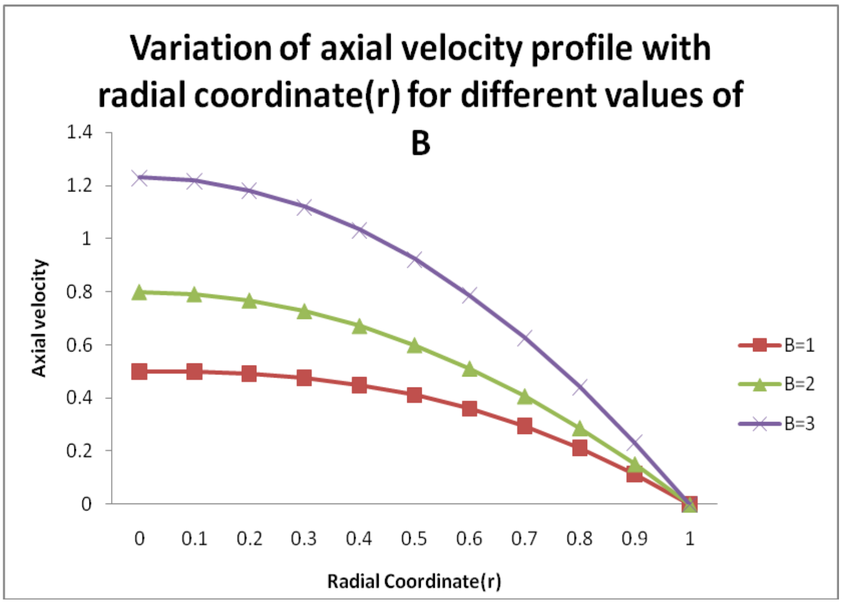

Figure-1

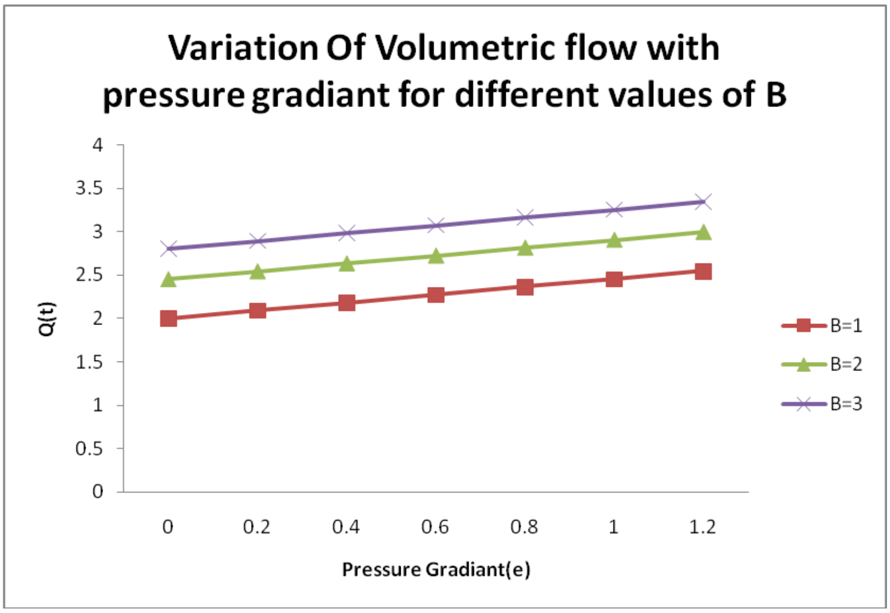

Figure-2

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